Chapter 6 / **Example 10**

Modelling with polynomials

<A short introductory text, where appropriate>

The path of a ski jumper is illustrated on the diagram on the right.
In order to determine a function to model this jump, the diagram was processed with graphing software to determine a set of points that the path of the ski jumper goes through, *x* being the horizontal distance from the starting point and *y* being the vertical displacement from the starting point. The points are listed in the table on the next page.

x	34.8	58.2	83.1	105.9	120.3	141.9	161.4	180	195	206.1	218.4	228.9	239.4
y	–19.5	–34.2	–49.5	–61.8	–64.8	–64.8	–68.1	–74.4	–82.5	–90.6	–101.1	–112.5	–125.7

a Enter the data into your GDC.
b What type of function would model this set of data points and why?
c Use your GDC to determine the model function for this set of data.
d Assess the choice of model by determining the coefficient of determination.
e Plot the model function over the scatter plot and comment on the closeness of fit to the original data.

The straight line that represents the slope of the mountain on which the skier will land goes through the points (0, 0) and (360, -210).

f Determine the equation of the line going through these two points.
g Find the point at which the ski jumper will land on the slope.

From that point on and up to the point where $x = 400$, the skier moves along the straight line describing the slope of the mountain.

h Write in piecewise form the function describing the whole path of the skier.

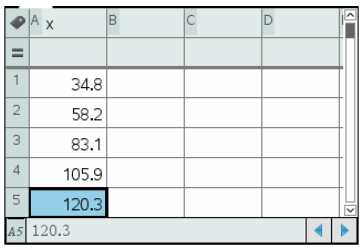
Open a new document and add a Lists & Spreadsheet page.

Type 'x' in the first cell.

Enter the *x*-coordinates in the first column.

Press enter or ▼ after each number to move to the next cell.

Note: *'x' is a label that will be used to calculate the regression equation. You can use any letter or name to label the list.*



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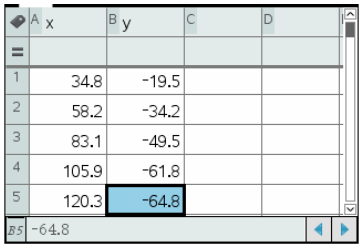
Type 'y' in the cell to the right of 'x'.

Enter the *y*-coordinates in the second column.

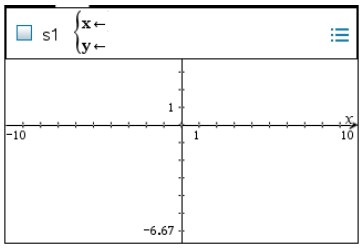
Use the ▲ ▼ ▶ ◀ keys on the touchpad to navigate the spreadsheet.



Add a new Graphs page to your document by pressing ctrl doc (+page) 2:Add Graphs.

Press menu 3:Graph Entry/Edit | 6:Scatter Plot.

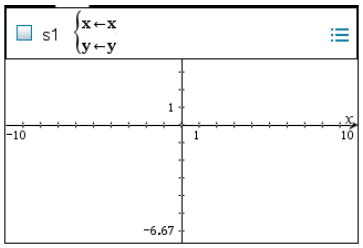
The entry line is shown at the top of the work area.



Type the names of the two variables you created in the spreadsheet: 'x' and 'y'.

You can also select these from the dropdown list by pressing var.

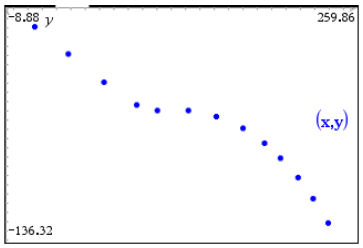
Press enter.



To view the data with suitable axes, press menu 4:Window/Zoom | 9:Zoom – Data.

The GDC displays a scatter diagram of *x* against *y*.

Because the data is approximately cubic, cubic regression is appropriate.



Return to the Lists & Spreadsheet page by pressing ctrl ◀ del

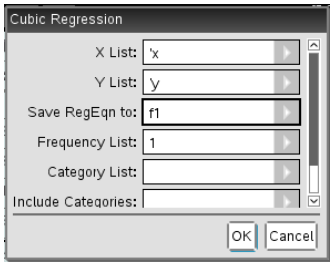
To calculate the equation of quadratic regression

Press menu 4:Statistics | 1:Stat Calculations | 7:Cubic Regression…

Open the drop down lists with ▶ and select using ▼ and enter del

Choose 'x' for X List, 'y' for Y List, f1 for Save RegEqn to and leave the remaining fields unchanged.

Click the touchpad on OK or press enter del



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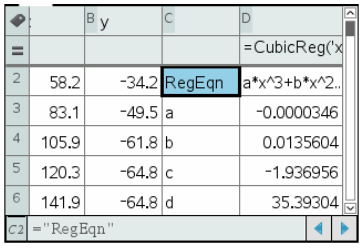
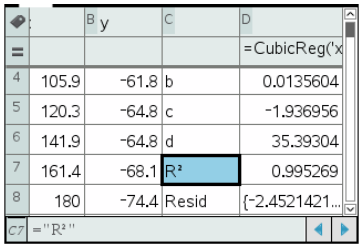
The cubic curve is given by the equation $y = -0.0000346\,x^3 + 0.0136\,x^2 - 1.94\,x + 35.4$.



Scroll down using ▼.

The coefficient of determination is $R^2 = 0.995$, which shows very strong cubic association.

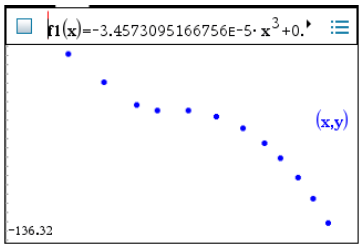


Return to the Graph page by pressing ctrl ▶ del

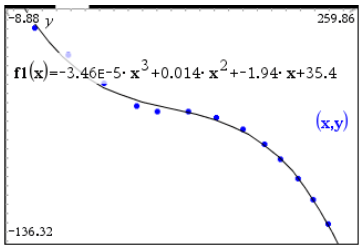
Press tab to display the entry line again.

Press menu 3:Graph Entry/Edit | 1:Function.

This time '*f2*(*x*)=' is displayed. Press ▲ to display '*f1*(*x*)' and press enter to select it.



The GDC displays the scatter diagram and the regression line.

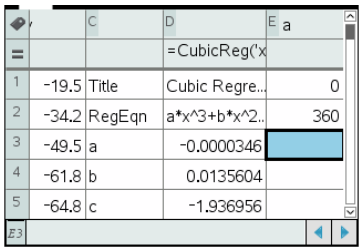


Return to the Lists & Spreadsheet page by pressing ctrl ◀ del

Move to the next available column and enter *a* in the first cell.

Enter the *x*-coordinates: 0 and 360 in the column.

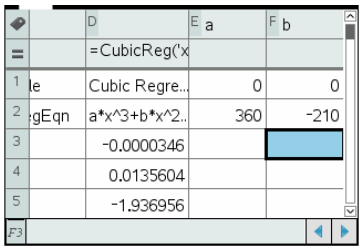
Press enter or ▼ after each number to move to the next cell.



Type 'b' in the cell to the right of 'a'.

Enter the *y*-coordinates: 0 and −210 in the second column.

Use the ▲ ▼ ▶ ◀ keys on the touchpad to navigate the spreadsheet.



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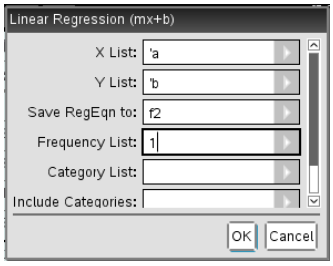
Press menu 4:Statistics | 1:Stat Calculations | 3:Linear Regression (mx+b)…

Open the drop down lists with ▶ and select using ▼ and enter del

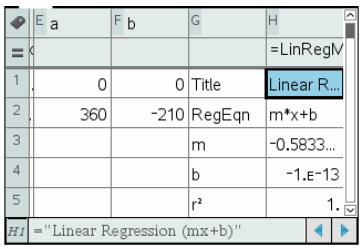
Choose 'a' for X List, 'b' for Y List, f2 for Save RegEqn to and leave the remaining fields unchanged.

Click the touchpad on OK or press enter del



The straight line is given by the equation $y = -0.0.583x$.

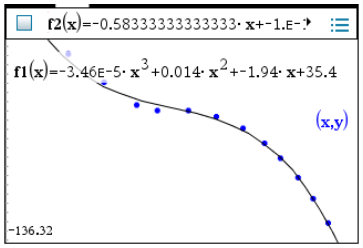
Ignore b = -1.E-13 as this is a very small number and is due to the numerical method that the calculator finds the regression equation.



Return to the Graph page by pressing ctrl ▶ del

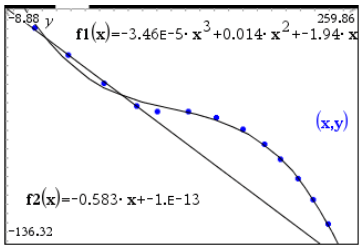
Press tab to display the entry line again.

This time '*f3*(*x*)=' is displayed. Press ▲ to display '*f2*(*x*)' and press enter to select it.



The GDC displays the two functions.

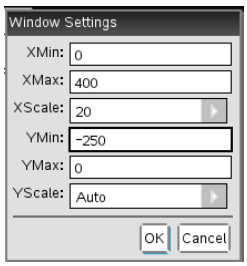
Change the window to show the later part of the skier's path.



Press menu 4:Window/Zoom | 1:Window Settings…

Set the axes to show $0 \le x \le 400$ and $-250 \le y \le 0$ with scales of 20.

Press enter when you have finished.



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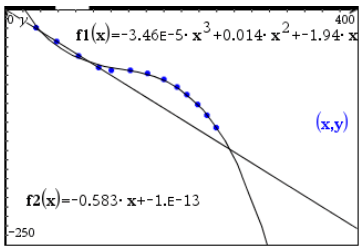
The GDC displays the two functions in a suitable window.

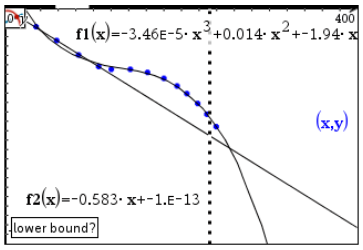


Press menu 6:Analyse Graph | 4:Intersection.

To find the intersection you need to give the lower and upper bounds of the region that includes the intersection.

The GDC shows a line and asks you to set the lower bound. Move the line using the touchpad and choose a position to the left of the intersection.

Click the touchpad.

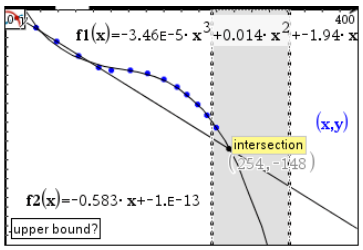


The GDC shows another line and asks you to set the upper bound.

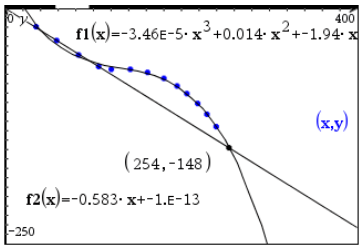
Use the touchpad to move the line so that the region between the lower and upper bounds contains the intersection.

When the region contains the intersection, the calculator will display the word 'intersection' in a box.

Click the touchpad.



The GDC displays the intersection of the curve and the straight line at the point $(254, -148)$.



The piecewise function is

$$f(x) = \begin{cases} -0.0000346x^3 + 0.0136x^2 - 1.94x + 35.4, & 0 \le x \le 254 \\ -0.583x, & 254 < x \le 400 \end{cases}$$

Press tab and modify $f1(x)$ by adding $| \, 0 \le x \le 254$ and $f2(x)$ by adding $| \, 254 < x \le 400$.

|, ≤ and < can both be found by pressing ctrl = ([|≠≥]) and choosing the symbols.